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INVESTIGATION OF THE STABILITY OF A TWO-BLADED ROTOR ON AN ANISOTROPIC BASE

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INVESTIGATION OF THE STABILITY OF A TWO-BLADED ROTOR ON AN ANISOTROPIC BASE†

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(The article was presented by P. M. Riz, Professor at the Moscow Automechanics Institute.)

ABSTRACT: The results of the effect of the most important parameters (coefficient of anisotropy of the base S; frequency of natural oscillations of the blade, caused by the flexibility in the vertical link P_{0b}) on the stability are examined in this paper. In the only study [1] treating this problem, Coleman drew an invalid conclusion which stated that, if the rigidities of the base in different directions do not differ too greatly from one another ($S \cong 1$), then we can consider the support isotropic. The results obtained here show the error in such a statement by Coleman.

SOME RESULTS OF AN ANALYSIS OF THE STABILITY

The oscillations of a two-bladed rotor on an anisotropic base are described by the following equation: /31*

$$\left.\begin{aligned}
&\ddot{x} + 2n_x\dot{x} + P_x^2 x + \varepsilon_1[(\ddot{\eta} - \omega^2\eta)\sin\omega t + 2\omega\dot{\eta}\cos\omega t] = 0,\\
&\ddot{z} + 2n_z\dot{z} + P_z^2 z + \varepsilon_1[(\ddot{\eta} - \omega^2\eta)\cos\omega t - 2\omega\dot{\eta}\sin\omega t] = 0,\\
&\ddot{\eta} + 2n_\lambda\dot{\eta} + (\omega^2\nu_0^2 + P_{0_b}^2)\,\eta + 2\varepsilon_2[\ddot{x}\sin\omega t + \ddot{z}\cos\omega t] = 0,
\end{aligned}\right\} \quad (1)$$

where x and z are the shifts of the rotor center toward the direction of the axes (Fig. 1 in [2]),

$$\eta = \xi_1 - \xi_2,$$

ξ_1 and ξ_2 are the angular deviations of the blades (Fig. 1 in [2]),

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† See the beginning in No. 3, 1967.

* Numbers in the margin indicate pagination in the foreign text.

$$P_x^2 = C_x/M, \qquad P_z^2 = C_z/M,$$

P_x and P_z are the frequencies of natural oscillations of the base toward the direction of the x and z axes,

$$2n_x = K_x/M, \qquad 2n_z = K_z/M,$$

C_x, C_z, K_x, K_z are the coefficients of rigidity and damping of the flexible base toward the direction of the x and z axes,

$$P_{\psi b}^2 = C_b/J_b, \qquad 2n_b = K_b/J_b,$$

P_{0b} is the frequency of natural oscillations of the blade, caused by the flexibility in the vertical link, /32

C_b and K_b are the angular coefficients of rigidity and damping of the blade in the vertical link,

J_b is the moment of inertia of the blade in relation to the vertical link,

$$\varepsilon_1 = \frac{m_b r}{m + 2m_b},$$

m_b is the mass of the blade,

r is the radius of the center of gravity of the blade, in relation to the vertical link,

M is the mass of the flexible base (reduced mass of the body),

$$\nu_0^2 = S_b l_{vl}/J_b,$$

S_b is the static moment of the mass of the blade, in relation to the vertical link,

l_{vl} is the loss of the vertical link,

$$\varepsilon_2 = r/\rho^2,$$

ρ is the radius of inertia for the blade, in relation to the vertical link.

Let us turn to the system in dimensionless coordinates. We will assume that $\omega t = \psi$.

$$\frac{d^2\bar{x}}{d\psi^2} + 2\bar{n}_x \frac{1}{\bar{\omega}} \frac{d\bar{x}}{d\psi} + \frac{1}{\bar{\omega}^2}\bar{x} + \bar{\varepsilon}_1 \left[\left(\frac{d^2\eta}{d\psi^2} - \eta \right) \sin\psi + 2\frac{d\eta}{d\psi}\cos\psi \right] = 0, \tag{2}$$

$$\left.\begin{aligned}&\frac{d^2\bar z}{d\psi^2}+2\bar n_z\frac{1}{\bar\omega}\frac{d\bar z}{d\psi}+S^2\frac{1}{\bar\omega^2}\bar z+\bar\varepsilon_1\left[\left(\frac{d^2\eta}{d\psi^2}-\eta\right)\cos\psi-\right.\\&\qquad\left.-2\frac{d\eta}{d\psi}\sin\psi\right]=0,\\&\frac{d^2\eta}{d\psi^2}+2\bar n_b\frac{1}{\bar\omega}\frac{d\eta}{d\psi}+\left(\nu_0^2+\bar P_{0b}^2\frac{1}{\bar\omega^2}\right)\eta+\\&\qquad+2\bar\varepsilon_2\left[\frac{d^2x}{d\psi^2}\sin\psi+\frac{d^2\bar z}{d\psi^2}\cos\psi\right]=0.\end{aligned}\right\}\quad(2\ \text{Cont.})$$

Here $x = \bar{x}/r$; $\bar{z} = z/r$; $\bar{n}_x = n_x/P_x$; $\bar{n}_z = n_z/P_x$; $\bar{n}_b = n_b/P_x$; $\bar{P}_{0b} = \bar{P}_{0b}/P_x$; and $S = P_z/P_x$ is the coefficient of anisotropy:

$$\bar\omega=\frac{\omega}{P_x};\qquad \bar\varepsilon_1=\frac{m_b}{M+2m_b};\qquad \bar\varepsilon_2=\left(\frac{r}{\rho}\right)^2;\qquad \varepsilon=\bar\varepsilon_1\bar\varepsilon_2.$$

System (2) is equivalent to the following system: /33

$$\left.\begin{aligned}&\dot y_1=y_4,\quad \dot y_2=y_5,\quad \dot y_3=y_6,\\&\dot y_4=\frac{1}{1-2\varepsilon}\left\{\frac{1}{\bar\omega^2}(2\varepsilon\cos^2\psi-1)\,y_1-\varepsilon\sin 2\psi_1\frac{S^2}{\bar\omega^2}y_2+\right.\\&\quad+\bar\varepsilon_1\sin\psi\left(1+\nu_0^2+\frac{\bar P_{0b}^2}{\bar\omega^2}\right)y_3+2\bar n_x\frac{1}{\bar\omega}(2\varepsilon\cos^2\psi-1)\,y_4-\\&\quad\left.-2\bar n_z\frac{1}{\bar\omega}\varepsilon\sin 2\psi\,y_5-2\bar\varepsilon_1\left[(1-2\varepsilon)\cos\psi-\bar n_b\frac{1}{\bar\omega}\sin\psi\right]y_6\right\},\\&\dot y_5=\frac{1}{1-2\varepsilon}\left\{-\frac{1}{\bar\omega^2}\varepsilon\sin 2\psi\,y_1-\frac{S^2}{\bar\omega^2}(1-2\varepsilon\sin^2\psi)\,y_2+\right.\\&\quad+\bar\varepsilon_1\cos\psi\left(1+\nu_0^2+\frac{\bar P_{0b}^2}{\bar\omega^2}\right)y_3-2\bar n_x\frac{\varepsilon}{\bar\omega}\sin 2\psi\,y_4-\\&\quad\left.-2\bar n_z\frac{1}{\bar\omega}(1-2\varepsilon\sin^2\psi)\,y_5+2\bar\varepsilon_1\left[(1-2\varepsilon)\sin\psi+\bar n_b\frac{1}{\bar\omega}\cos\psi\right]y_6\right\},\\&\dot y_6=\frac{1}{1-2\varepsilon}\left\{2\bar\varepsilon_2\frac{1}{\bar\omega^2}\sin\psi\,y_1+2\bar\varepsilon_2\frac{S^2}{\bar\omega^2}\cos\psi\,y_2-\left(\nu_0^2+2\varepsilon+\frac{\bar P_{0b}^2}{\bar\omega^2}\right)\times\right.\\&\quad\left.\times y_3+4\bar\varepsilon_2\bar n_x\frac{1}{\bar\omega}\sin\psi\,y_4+4\bar\varepsilon_2\bar n_z\frac{1}{\bar\omega}\cos\psi\,y_5-2\frac{\bar n_b}{\bar\omega}y_6\right\},\\&y_1=\bar x,\quad y_2=\bar z,\quad y_3=\eta,\quad y_4=d\bar x/d\psi,\quad y_5=d\bar z/d\psi,\quad y_6=d\eta/d\psi.\end{aligned}\right\}\quad(3)$$

System (3) examines the stability according to the method mentioned in a previous article [2]. All the calculations were made on an electron computer M-20. For this purpose, we made up a special program.

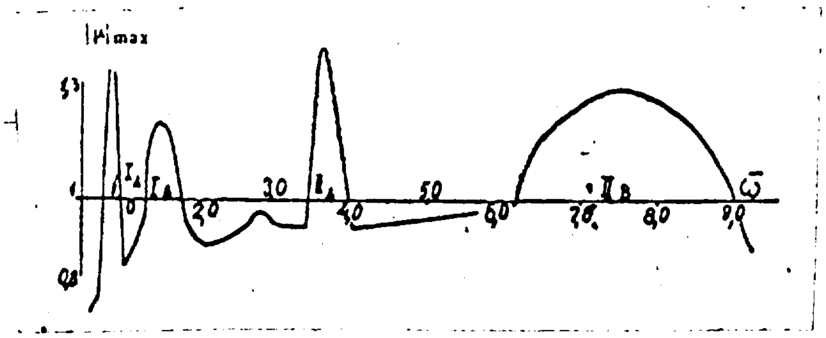

Fig. 1. Graph of the Dependence of $|\mu|_{max}$ on the Relative Convolutions of the Rotor $\overline{\omega} = \omega/D_x$, $S = 4$.

The behavior of $|\mu|_{max}$ within the range for a change in the relative convolutions of a rotor $\overline{\omega} \in (0.5; 9.1)$, with $S = 4$, is shown in Figure 1. There are two pairs of ranges (zones) of instability. We will call the first pair of zones I_A and I_B, and the second pair /34
II_A and II_B. If we are interested in only one pair, then we will call the zones A and B. In the I_A and II_A zones it was found that $Jm\mu = 0$. These zones will be called aperiodic instability zones. In the I_B and II_B zones, $Jm\mu \neq 0$. These zones will be called oscillational instability zones. In the I_A and I_B zones, oscillations occur along the x-axis. In the II_A and II_B zones, they occur along the z-axis.

(a) Effect of the Anisotropy Coefficient of the Base on the Stability. We are examining the effect of S on the instability zones during a decrease in S from 3 to 1. It was assumed that, as S approaches 1, the two pairs of zones above will converge or, more accurately, the second pair of zones (II_A and II_B) will approach the first pair (I_A and I_B), and will combine with it when $S = 1$. But it was found that, with values of S close to 1, the picture becomes more complicated. A certain additional zone D makes its appearance. There comes a moment when, in the interval of practical convolutions of the rotor $\overline{\omega} \in (0; 5; 3.0)$, there are five entire, fairly expansive, instability zones ($S = 1.4$), Figure 2.

Zone D occurs at $S \approx 2.2$, isolated from the right of the II_A zone. With a further decrease in S, the D zone breaks completely from the II_A zone. Thus $Jm\mu \neq 0$ in the D zone. Therefore, we will also call it an oscillational instability zone. While the II_A and I_B zones approach the I_A zone, as we would anticipate, the height H and the width $\Delta\overline{\omega}$ of these zones (II_A and I_B) decrease significantly; but the II_A zones increase, as we can also see from the Table (the lower number in the boxes is the value of $\Delta\omega$ multiplied by 10^2, and the upper number is the value of H, multiplied by 10^4).

Thus, we achieve the greatest H (zone A-7766, zone B-2468) and width of the zone $\Delta\overline{\omega}$ (zone A - 28, zone B - 90) when S = 1.

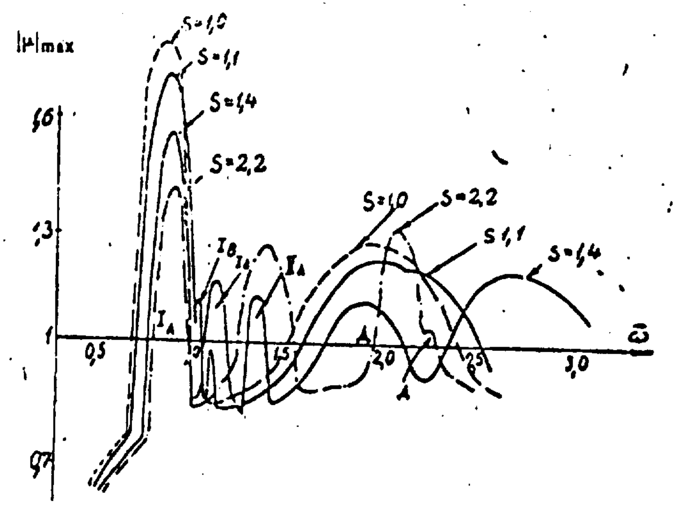

Fig. 2. Graph of the Greatest Multiplier (by Modulus) of $|\mu|_{max}$ Versus the Anisotropy Coefficient for a Value of $S = P_z/P_x$ Close to 1. $\overline{n}_x = \overline{n}_z = n_b = 0.05$; $\nu_0 = 0.3$; $\varepsilon = 0.05$; $\overline{P}_{0b} = 0$; $\overline{\varepsilon}_2 = 0.894$; $\Delta\psi = 2\pi/100 = 3.6°$.

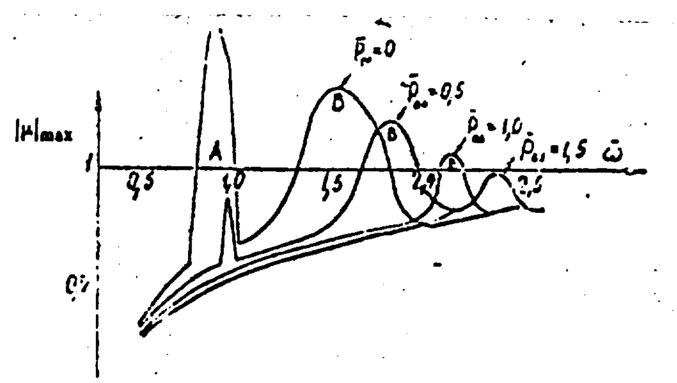

Fig. 3. Graph of the Greatest Multiplier (by Modulus) of $|\mu|_{max}$ Versus the Relative Natural Frequencies of the Blade in the Vertical Link $\overline{P}_{0b}$. $\varepsilon = 0.05$; $\overline{n}_x = \overline{n}_z = n_b = 0.05$; $\nu_0 = 0.3$; $\varepsilon_2 = 0.894$; $S = 4$.

(b) Effect of Relative Frequency of Natural Oscillations of a Blade, Caused by the Flexibility in the Vertical Link $\overline{P}_{0b}$, on the Stability (Fig. 3). With an insignificant increase in $\overline{P}_{0b}$, zone A disappears. In this case, B shifts to the right, and the amplitude and width of zone B decrease significantly. When P_{0b} is /35 equal to 1.5, zones A and B disappear completely. These calculations were made for damping coefficients of $\overline{n}_x = \overline{n}_z = \overline{n}_b = 0.05$.

S / H	3	1,8	1,2	1
I_A	3735 / 18	4402 / 20	6767 / 24	7766 / 28
I_B	2261 / 40	2201 / 20	1241 / [illegible]	0 / 0
II_A	3592 / 19	2235 / 18	721 / 7	0 / 0
Д	— / —	722 / 38	1446 / 46	
II_B	—	—	1683 / 74	

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